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Regional Drought Distribution Model

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Abstract. A model for regional drought distribution analysis is presented. This model is based on a multivariate distribution of an instrumental variable and enables one to analyse the spatial distribution of droughts and associated risks, expressed by the exceedance probability.

The instrumental variables series are associated to elementary areas of the region and simulated series are obtained from a synthetic simulation model. The results obtained from the regional drought analysis of the simulated series allows the development of severity-area-frequency curves and the analysis of the regional drought return periods of the historical series.

The application of the model to the annual precipitation over the area of the Guadiana river basin, in Portugal, for the 1940/41-1994/95 period suggests that most droughts have a duration of one year. The most severe drought, in the hydrological year 1944/45, has an associated return period of 100 years and the droughts observed in 1980/81, 1982/83 and 1994/95 have associated return periods greater than 25 years.

1 Introduction

Droughts are long term phenomena affecting large regions leading to important losses in most social and economic activities. Droughts are defined as an exceptional event characterised by insufficient water resources availability for supplying the socio-economic activities of the region.

The areal distribution of drought is a significant characteristic of these phenomena; thus the regional approach leads to a more comprehensive evaluation of this situation.

Methods of analysing regional drought are not common. A regional method of drought characterisation based on the theory of runs and its application to Portuguese precipitation is described by Santos (1981; 1983). Rossi (1983) presents deficit-area-frequency curves for the whole Sicily based on the statistical analysis of precipitation deficits. A method for regional frequency analysis of droughts and its application to flows in California is presented by Sadeghipour and Dracup (1985). However, using these methods, the areal distribution of droughts is not sufficiently identified, thus the drought areas can not be adequately established.

The regional drought distribution model presented in this paper is based on a hydrological instrumental variable associated to a time period. Instrumental variables are associated to the region under study, to a threshold drought level and to a critical drought area established. Drought severity, drought area and drought duration are output variables of the model. Return periods of regional droughts determination is based on the spatial evolution of drought severity.

The proposed methodology for regional drought characterisation enables the clear identification of drought areas, the simulation of the spatial distribution of droughts and the associated risk of their occurrence.

2 Model description

Considering a region, for instance a river basin or a set of river basins, the analysis of droughts can be focused on the precipitation (the instrumental variable) that occurred during a given interval (one year, semester or month). To characterise the drought event, in this case, it is necessary to get a set of precipitation series of the study region and to associate to each precipitation station an influence area.

Let X_{ij} be the instrumental variable in the area i during the time period j; the regional drought parameters depend on the marginal distribution of X_{ij}, the temporal dependence of X_{ij} and on the spatial dependence between X_{ij} for different locations. Assuming M to be the total number of series of the instrumental variable, and A_i the area influenced by each series, the total area AR is:

$$AR = \sum_{i=1}^{M} A_i \qquad (1)$$

If the variables X_{ij} have the same marginal distribution, except for a location parameter μ_i, a scale parameter σ^2_i and a shape parameter λ_i, the distribution of Z_{ij} obtained from X_{ij} using an appropriate transformation, is independent from i and j. It is considered that the standardised normal distribution fits the distribution of Z_{ij}

which is obtained by the Box-Cox transformation for the skew correction of $X_{i,j}$:

$$W_{i,j} = \frac{X_{i,j}^{\lambda_i} - 1}{\lambda_i} \qquad (2)$$

and by transforming linearly the variables $W_{i,j}$:

$$Z_{i,j} = \frac{W_{i,j} - \mu_i}{\sigma_i} \qquad (3)$$

It is accepted that the spatial dependence of $X_{i,j}$ is well represented by the spatial correlation of $Z_{i,j}$, considering two locations i (p and q) in the analysed time period:

$$\rho^{(p,q),N} = \frac{E\left[Z_{p,N} Z_{q,N}\right]}{\sqrt{E\left[Z_{p,N}^{2}\right] E\left[Z_{q,N}^{2}\right]}} \qquad (4)$$

Drought occurs when the value of the instrumental variable is less than a fixed threshold. The threshold $\mathcal{L}$ represents a limit value of water use in a region. Drought is defined in the time period j, when the standardised variable $Z_{i,j}$, associated to an area A_i is lower than $\mathcal{L}$. The duration of the drought initiated in period j (D_j) is the consecutive number of time periods where $Z_{i,j}$ is lower than $\mathcal{L}$ in an area greater than a critical area considered.

The average drought area and the average drought severity of the drought initiated in period j, with duration D_j, are defined as:

$$\overline{As_{j,D_j}} = \frac{\sum_{\ell=j}^{j+D_j-1} As_\ell}{D_j}, \quad \overline{S_{j,D_j}} = \frac{\sum_{\ell=j}^{j+D_j-1} S_\ell}{D_j} \qquad (5),(6)$$

It is also possible to define the maximum drought area and the maximum drought severity of the drought initiated in the time period j, with duration D_j:

$$As_{j,D_j}^{max} = \max_{\ell=j,j+D_j-1} (As_\ell) \qquad (7)$$

$$S_{j,D_j}^{max} = \max_{\ell=j,j+D_j-1} (S_\ell) \qquad (8)$$

The model for drought analysis in each time period is presented in Fig. 1. In a first step the region's elementary area where $Z_{i,j}$ is smaller is selected. Being $Z_{i,j}$ lesser than $\mathcal{L}$ drought is defined in the time period analysed. Drought area $As^{(k)}$, standard transformed variable $W^{(k)}$, average $\overline{W^{(k)}}$ and standard deviation $s^{(k)}$ are the ones verified in the first area selected:

$W^{(k)} = W_{i(k),j}$, $\overline{W^{(k)}} = \overline{W}_{i(k)}$, $s^{(k)} = s_{i(k)}$. Indices sign the calculation steps.

For the second drought calculation step a second area is selected, adjacent to the first one, where the standard variable $Z^{(k)}$ in the total area $As^{(k)}$ is smaller:

$$Z^{(k)} = \frac{var_1 + var_2}{\sqrt{var_3 + var_4 + var_5}} \qquad (9)$$

where:

$$var_1 = As^{(k-1)} s^{(k-1)} Z^{(k-1)}$$

$$var_2 = A_{i(k)} s_{i(k)} Z_{i(k),j}$$

$$var_3 = \left(As^{(k-1)}\right)^2 \left(s^{(k-1)}\right)^2$$

$$var_4 = A_{i(k)}^{\ 2} s_{i(k)}^{\ 2}$$

$$var_5 = 2\rho^{(k-1,i(k))} As^{(k-1)} A_{i(k)} s^{(k-1)} s_{i(k)}$$

The correlation coefficient between standardised transformed variables in the area $As^{(k)}$ and the selected area i, $\rho^{(k-1,i(k))}$, is obtained from:

$$\rho^{(k-1,i(k))} = \frac{\sum_{j=1}^{N} var_6 \, var_7}{\sqrt{\sum_{j=1}^{N} var_6^{\ 2} \sum_{j=1}^{N} var_7^{\ 2}}} \qquad (10)$$

where:

$$var_6 = Z_j^{(k-1)} - \left(\sum_{j=1}^{N} Z_j^{(k-1)} / N\right)$$

$$var_7 = Z_{i(k),j} - \left(\sum_{j=1}^{N} Z_{i(k),j} / N\right)$$

$As^{(k)} = As^{(k-1)} + A_{i(k)}$ is the drought area. The variable's value is:

$$X^{(k)} = \frac{\sum_{\ell=1}^{k-1} A_{i(\ell)} X_{i(\ell),j} + A_{i(k)} X_{i(k),j}}{As^{(k-1)} + A_{i(k)}} \qquad (11)$$

The mean $\overline{W^{(k)}}$ and the standard deviation $s^{(k)}$ of the standard transformed variable are obtained:

$$\overline{W^{(k)}} = \frac{As^{(k-1)} \overline{W^{(k-1)}} + A_{i(k)} \overline{W_{i(k)}}}{As^{(k-1)} + A_{i(k)}} \qquad (12)$$

$$s^{(k)} = \frac{\sqrt{var_3 + var_4 + var_5}}{As^{(k-1)} + A_{i(k)}} \qquad (13)$$

where var_3, var_4 and var_5 are defined in the Eq. 9.

The calculation proceeds by introducing in the sequent step (k+1) the values of the adjacent area i that results in a higher $-Z^{(k+1)}$. The calculation of $As^{(k+1)}$, $Z^{(k+1)}$, $X^{(k+1)}$, $\overline{W^{(k+1)}}$ and $s^{(k+1)}$ is similar. Selected areas are associated to the standardised transformed variable in each time interval. The standardised transformed variable Z represents the drought severity.

Drought duration, average drought area, average drought severity, maximum drought area and maximum drought severity calculation result from the analysis of the standardised transformed variable calculated in the total drought area for the different drought time periods.

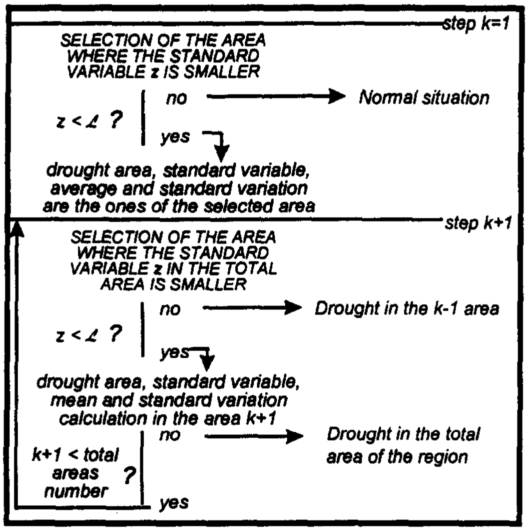

Fig. 1 Model for drought analysis in each time period.

3 Drought analysis

3.1 Regional drought analysis sequence

Regional drought analysis is based on recorded data series and constrains the analysis of hydrological time series in an identical time period. The number of droughts observed is often reduced in order to study drought return periods. To overcome this one can simulate instrumental variable series preserving some of their statistical characteristics, so that several values sequences and consequently several drought episodes can be obtained.

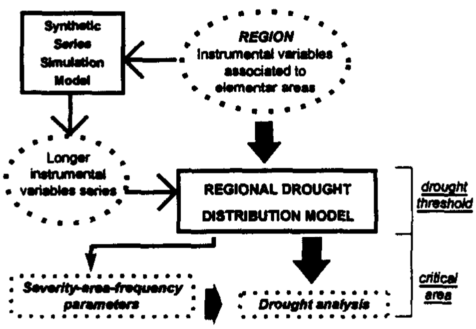

Fig. 2 Regional drought analysis sequence.

The regional drought analysis sequence is represented in Fig. 2. Based on the instrumental variables series associated to elementary areas of a region and the simulated series obtained from the synthetic simulation model, the regional drought distribution model is applied. The results obtained from the simulated series allows the development of severity-area-frequency curves and the analysis of the regional drought return periods of the historical series.

3.2 Multivariate simulation model

The multivariate simulation model considered preserves means (μ_i) and the variances (σ_i^2) of $X_{i,j}$, the regional Box-Cox parameter (λ) and the covariances between $X_{i,j}$. The transformed variables $W_{i,j}$ using the regional Box-Cox parameter λ have the joint multivariate normal distribution:

$$f_{X^{(\lambda)}}(\underline{W}^{(\lambda)};\underline{\mu},\underline{\Sigma}) = \frac{1}{(2\pi)^{M/2}\sqrt{|\underline{\Sigma}|}} \times \exp\left[-\frac{1}{2}(\underline{W}^{(\lambda)}-\underline{\mu})^T \underline{\Sigma}^{-1}(\underline{W}^{(\lambda)}-\underline{\mu})\right] \qquad (14)$$

being $\underline{W}^{(\lambda)}$ the vector of the transformed variables $W_{i,j}$ and $\underline{\mu}$ the vector of the means μ_i of the transformed variables.

The calculation of the variance-covariance matrix $\underline{\Sigma}$ (M×M elements) is based on the linear correlation between the historical transformed series matrix $\underline{R}$:

$$\underline{\Sigma} = \underline{S}\,\underline{R}\,\underline{S} \qquad (15)$$

$\underline{S}$ is the diagonal matrix whose elements are the standard deviation values s_i. Considering the vector of random variables with normal standard distribution $\underline{\varepsilon}$, the vector $\underline{W}^S$ defined:

$$\underline{W}^S = \underline{C}\,\underline{\varepsilon} + \overline{W} \qquad (16)$$

is normal distributed with mean:

$$\overline{W} = \begin{bmatrix} \overline{W}_1 \\ \vdots \\ \overline{W}_M \end{bmatrix}$$

and variance-covariance matrix $\underline{\Sigma} = \underline{C}\,\underline{C}^T$.

3.3 Severity-area-frequency curves

Drought severity is analysed based on severity-area-frequency curves calculated for the periods more affected by drought selected from simulated precipitation series. Drought severity is represented by the transformed standardised variable ($-Z^{(k)}$). The pairs of the transformed standardised variable and affected areas in the worst drought periods are obtained using the regional drought distribution model. Severity values in each area are ordered to associated return periods T. In case of unequal elementary areas (if the instrumental variable series are represented by different areas) study areas should be defined (e.g. 10,20,...,100% of the total area).

Return periods are defined using an empirical distribution function adequate to the studied drought and an appropriate distribution model.

3.4 Droughts in the Portuguese Guadiana river basin

The Portuguese Guadiana river basin is located in the south, covering a total area of 11525 km^2. The annual precipitation is analysed, in the 1940/41-1994/95 period, and 35 stations are analysed. The Thiessen method was used to evaluate the influence areas of precipitation.

The drought threshold was selected to correspond to a level of 80% of the exceedance probability. The regional Box-Cox parameter is approximately calculated by $\lambda = \left(\prod_{i=1}^{M} \lambda_i\right)^{1/M} = 0.405$. This is very close to the partial maximum likelihood estimator of the regional λ.

The multiyear analysis suggests that most droughts in the region have a one year duration. Two droughts with two years duration (drought initiated in 1943/44 and in 1956/57) are verified in the period 1940/41-1994/95. Based on the average drought severity analysis, considering a critical area of 75% of the total area, a return period of 400 years is calculated for the drought initiated in 1943/44 and a return period of 13 years is calculated for the drought initiated in 1956/57.

Drought severity-area-frequency calculation is based on ten replicates with 100 years length of simulated precipitation. The average curve obtained from the ten replicates considers the 18 worst regional droughts. Severity-area-frequency curves are obtained by considering the extreme-value type 1 distribution:

$$-Z(As) = u(As) + \alpha(As)\,(-\log(-\log(1-1/T)) \qquad (17)$$

Severity-area-frequency curves for the Portuguese Guadiana river basin and the most important droughts in the region (T>5 years) are presented in Fig. 3. In the Guadiana basin the most severe drought, in the hydrological year 1944/45, has an associated return period of 100 years. Droughts observed in 1980/81, 1982/83 and 1994/95 have associated return periods greater than 25 years.

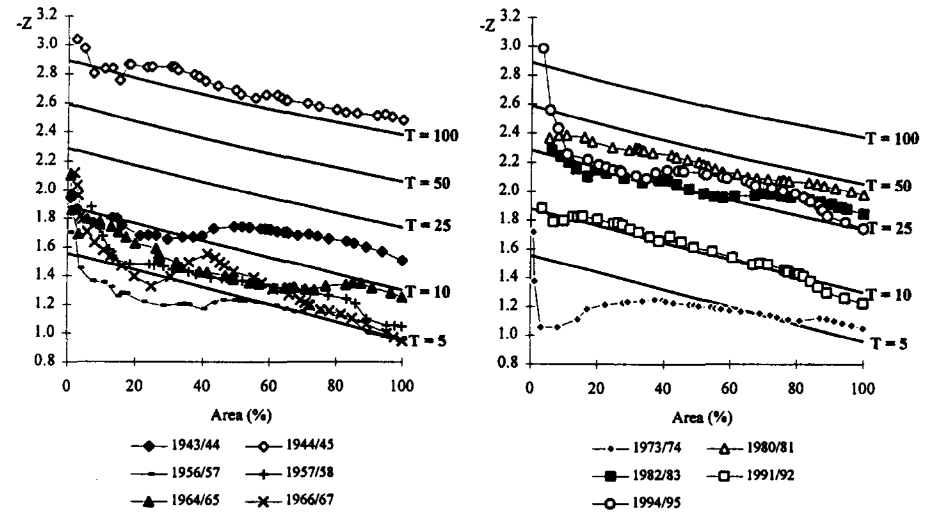

Fig. 3 Severity-area-frequency curves and return period (T) of the most important droughts in the 1940/41-1993/94 period in the Portuguese Guadiana river basin.

References

Henriques, A.G. 1990 - *Modelos de distribuição de frequências de caudais de cheia*, PhD Thesis, Universidade Técnica de Lisboa, Instituto Superior Técnico, Lisbon.

Henriques, A.G. and Santos, M.J.J. 1996 - Modelo de caracterização de secas regionais, 3° Congresso da Água, Associação Portuguesa dos Recursos Hídricos/Associação Brasileira de Engenharia Sanitária e Ambiental, Lisbon, Vol. *II*, pp.49-59.

Rossi, G. 1983 - Droughts of Sicily, *Coping With Droughts*, Water Resources Publications, U.S.A., pp.244-258.

Rossi, G.; Benedini M.; Tsakiris G.; and Giakoumakis S. 1992 - Review article: On regional drought estimation and analysis, *Water Resources Management*, Vol.*6*, pp.249-277.

Sadeghipour, J. and Dracup, J.A. 1985 - Regional frequency analysis of hydrological multiyear droughts, *Water Resources Bulletin*, American Water Resources Association, Vol..*21*, n°3, pp.481-487.

Santos, M.A. 1981 - *On the stochastic characterisation of regional droughts*, Thesis, Laboratório Nacional de Engenharia Civil, Lisbon.

Santos, M.A. 1983 - Regional Droughts: A stochastic characterisation, *Journal of Hydrology*, Vol.*66*, pp.183-211.

Santos, M.J.J. 1996 - *Modelo de distribuição de secas regionais*, MSc Thesis, Universidade Técnica de Lisboa, Instituto Superior Técnico, Lisbon.